**Dual Problem:**

In mathematical optimization theory, duality or the duality principle is the principle that optimization problems may be viewed from either of two perspectives, the primal problem, or the dual problem. Consider a primal problem in a standard form and its objective is maximization then its dual is minimization problem. The dual uses the same parameters that are present in the primal problem, but in different places.

To explain in detail, let us consider the example of simple primal solution problem,

One numerical example of the maximization LP problem we are doing in class is

Maximize Z = 3x1 + 5x2

Subject to:

x1 ≤ 4

2x2 ≤ 12

3x1 + 2x2 ≤ 18

x1, x2≥ 0

then its corresponding dual problem will be,

Minimize C = 4y1 + 12y2 + 18y3

Subject to

y1 + 3y3 ≥ 3

2y2 + 2y3 ≥ 5

y1, y2, y3 ≥ 0

We can see from above that the resource values became coefficients to objective function and coefficients of constraints have been transposed.

**Interpretation of Dual:**

From the above equations we can see that primal problem has 2 decision variables and 3 constraints, but when it comes to dual problem there are 3 decision variables and 2 constraints.

In real time if it is applied to an industry,

* Assuming primal problem objective is to maximize profit of two products manufactured.
* The decision variables for the dual problem represent the price paid for one hour of that resource.
* Thus, the objective of dual problem will be minimizing costs.
* The simplex method can be applied to either problem to get us the solutions which can be used by the business.

**Shadow Prices:**

From the above equations of primal and dual problem,

The solution for primal is Z= 36 with (2,6) values as decision variables

The solution for dual is C= 36 with (0,1.5,1) values for y1, y2, y3

* The first constraint of the primal is 4, second is 12 and the third is 18.
* If we increase first constraint by 1, there is no effect i.e., Z=36
* Similarly, if we increase second and third constraints by 1, Z becomes 37.5 and 37 respectively.
* There is an increase of 1.5 and 1 units with increase of 1 unit in the resources.
* These are called the shadow prices of the primal problem which we can see that they are the solutions of Dual problem.
* Shadow price is defined as the rate at which optimal value Z increases with increase of resource values in the constraints.

**Sensitivity Analysis:**

Different constraints have varying degrees of ability to influence the objective function. Different resources don't affect an objective function equally. Some factors are more crucial than others to produce products and services.

There are mostly three kinds of sensitivity analysis that can be done,

* **Changes in the Coefficients of a Non basic Variable:**Suppose that the changes made in the original model occur in the coefficients of a variable that was non basic in the original optimal solution. We need to analyze whether the optimal solution is feasible to the given coefficients.
* **Introduction of a New Variable:**In some situations, if the business wants to add new activities to the problem, this amounts to adding new decision variable to the objective function. We will be analyzing the impact based on the addition of new decision variable to the entire optimal solution.
* **Changes to Resources:**As discussed above in shadow pricing, Sensitivity is the responsiveness of the objective function to one unit change in one resource

**Impact on Business Decision Making based on Dual:**

* As discussed above, based on the requirements companies want to make changes to the existing problem.
* If the company decides to increase a resource availability, the corresponding shadow price is calculated.
* These shadow prices can be easily calculated by solving the dual problem of its primal.
* Based on the shadow prices, companies can decide which constraint resources needs to be increased to increase the objective.
* Thus companies can get insights on each constraint and their behavior with the help of its dual problem to make a decision for a problem with changes.

References:

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